AP Environmental Science Math Prep

This year in APES you will hear the two words most dreaded by high school students...NO CALCULATORS! That's right, you cannot use a calculator on the AP Environmental Science exam. Since the regular tests you will take are meant to help prepare you for the APES exam, you will not be able to use calculators on regular tests all year either. The good news is that most calculations on the tests and exams are written to be fairly easy calculations and to come out in whole numbers or to only a few decimal places. The challenge is in setting up the problems correctly and knowing enough basic math to solve the problems. With practice, you will be a math expert by the time the exam rolls around. So bid your calculator a fond farewell, tuck it away so you won't be tempted, and start sharpening your math skills!

DO NOT PRINT OUT THESE PAGES. ONLY THE ANSWER SHEET!

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Reminders
1. Write out all your work, even if it's something really simple. This is required on the APES exam so it will be required on all your assignments, labs, quizzes, and tests as well.
2. Include units in each step. Your answers always need units and it's easier to keep track of them if you write them in every step.
3. Check your work. Go back through each step to make sure you didn't make any mistakes in your calculations. Also check to see if your answer makes sense. For example, a person probably will not eat 13 million pounds of meat in a year. If you get an answer that seems unlikely, it probably is. Go back and check your work.

Directions
Read each section below for review. Look over the examples and use them for help on the practice problems. When you get to the practice problems, write out all your work and be sure to include units on each step. Check your work.

Decimals
Part I: The basics
Decimals are used to show fractional numbers. The first number behind the decimal is the tenths place, the next is the hundredths place, the next is the thousandths place. Anything beyond that should be changed into scientific notation (which is addressed in another section.)

1/10 (Tenths)
1/100 (Hundredths)
1/1000 (Thousandths)

17.591

Part II: Adding or Subtracting Decimals
To add or subtract decimals, make sure you line up the decimals and then fill in any extra spots with zeros. Add or subtract just like usual. Be sure to put a decimal in the answer that is lined up with the ones in the problem.

$$123.0000$$
$$0.0079$$
$$+43.5000$$
$$\overline{166.5079}$$

$$27.583$$
$$-\ 0.200$$
$$\overline{27.383}$$

Part III: Multiplying Decimals

Line up the numbers just as you would if there were no decimals. DO NOT line up the decimals. Write the decimals in the numbers but then ignore them while you are solving the multiplication problem just as you would if there were no decimals at all. After you have your answer, count up all the numbers behind the decimal point(s). Count the same number of places over in your answer and write in the decimal.

$$3.77 \times 2.8 = ?$$

$$3.77 \ (2\ decimal\ places)$$
$$\times \ \ 2.8 \ (1\ decimal\ place)$$
$$\overline{3016}$$
$$+754$$
$$\overline{10.556} \ (3\ decimal\ places)$$

Part IV: Dividing Decimals

Scenario One: If the divisor (the number after the / or before the $\overline{}$) does not have a decimal, set up the problems just like a regular division problem. Solve the problem just like a regular division problem. When you have your answer, put a decimal in the same place as the decimal in the dividend (the number before the / or under the   ).

$$\overline{)}$$

```
          424.9
    38 ) 16146.2
         152
         ____
          44
          16
         ____
          186
          152
         ____
          342
          342
         ____
            0
```

Scenario Two: If the divisor does have a decimal, make it a whole number before you start. Move the decimal to the end of the number, then move the decimal in the dividend the same number of places.

$$3.8 \ )\ \overline{1614.62}$$

Then solve the problem just like a regular division problem. Put the decimal above the decimal in the dividend. (See Scenario One problem).

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

1. 1.678 + 2.456 =
2. 1256.93 x 12.38 =
3. 64.5 / 5 =
4. 114.54 / 34.5 =
5. 3300.584 / 34.67 =

Averages

To find an average, add all the quantities given and divide the total by the number of quantities.

> *Example:* Find the average of 10, 20, 35, 45, and 105.
>
> *Step 1: Add all the quantities.* 10 + 20 + 35 + 45 + 105 = 215
>
> *Step 2: Divide the total by the number of given quantities.* 215 / 5 = 43

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

6. Find the average of the following numbers: 11, 12, 13, 14, 15, 23, and 29
7. Find the average of the following numbers: 124, 456, 788, and 343
8. Find the average of the following numbers: 4.56, .0078, 23.45, and .9872

Percentages

Introduction:

Percents show fractions or decimals with a denominator of 100. Always move the decimal TWO places to the right go from a decimal to a percentage or TWO places to the left to go from a percent to a decimal.

> *Examples:* .85 = 85%.   .008 = .8%

Part I: Finding the Percent of a Given Number

To find the percent of a given number, change the percent to a decimal and MULTIPLY.

> *Example: 30% of 400*
>
> *Step 1:* 30% = .30
>
> *Step 2:*   400
>
>   x .30
>
>   12000
>
> *Step 3: Count the digits behind the decimal in the problem and add decimal to the answer.*
>
>   12000 → 120.00 → 120

Part II: Finding the Percentage of a Number

To find what percentage one number is of another, divide the first number by the second, then convert the decimal answer to a percentage.

> *Example:* What percentage is 12 of 25?
>
> *Step 1:* 12/25 = .48
>
> *Step 2:* .48 = 48% (12 is 48% of 25)

Part III: Finding Percentage Increase or Decrease

To find a percentage increase or decrease, first find the percent change, then add or subtract the change to the original number.

> *Example:* Kindles have dropped in price 18% from $139. What is the new price of a Kindle?
>
> *Step 1:* $139 x .18 = $25
>
> *Step 2:* $139 - $25 = $114

To find a total value, given a percentage of the value, DIVIDE the given number by the given percentage.

> *Example:* If taxes on a new car are 8% and the taxes add up to $1600, how much is the new car?
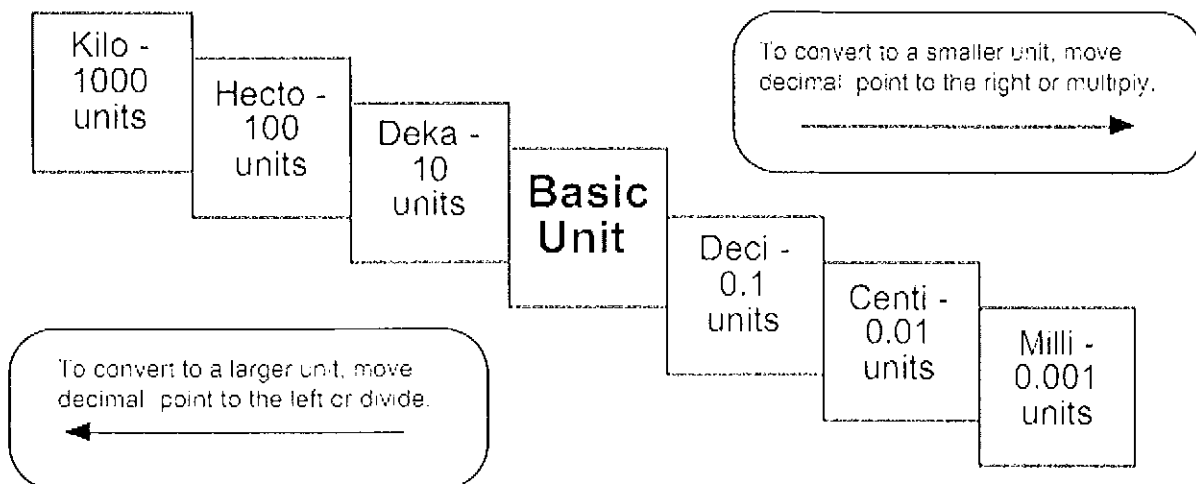>
> > *Step 1:* 8% = .08
> >
> > *Step 2:* $1600 / .08 = $160,000 / 8 = $20,000 *(Remember when the divisor has a decimal,*
> > *move it to the end to make it a whole number and move the decimal in the dividend the same*
> > *number of places. .08 becomes 8, 1600 becomes 160000.)*

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

9. What is 45% of 900?
10. A water heater tank holds 280 gallons. Two percent of the water is lost as steam. How many gallons remain to be used?
11. What percentage is 25 of 162.5?
12. 35 is what percentage of 2800?
13. Home prices have dropped 5% in the past three years. An average home in Indianapolis three years ago was $130,000. What's the average home price now?
14. The Greenland Ice Sheet contains 2,850,000 cubic kilometers of ice. It is melting at a rate of .006% per year. How many cubic kilometers are lost each year?
15. In a small oak tree, the biomass of insects makes up 3000 kilograms. This is 4% of the total biomass of the tree. What is the total biomass of the tree?

Metric Units

Kilo-, centi-, and milli- are the most frequently used prefixes of the metric system. You need to be able to go from one to another without a calculator. You can remember the order of the prefixes by using the following sentence: *King Henry Died By Drinking Chocolate Milk*. Since the multiples and divisions of the base units are all factors of ten, you just need to move the decimal to convert from one to another.

Kilo - 1000 units

Hecto - 100 units

Deka - 10 units

Basic Unit

Deci - 0.1 units

Centi - 0.01 units

Milli - 0.001 units

To convert to a smaller unit, move decimal point to the right or multiply. →

To convert to a larger unit, move decimal point to the left or divide. ←

Example: 55 centimeters = ? kilometers

> *Step 1:* *Figure out how many places to move the decimal. King Henry Died By Drinking... – that's six places. (Count the one you are going to, but not the one you are on.)*
>
> *Step 2:* *Move the decimal five places to the left since you are going from smaller to larger.*
>
> > *55 centimeters = .00055 kilometers*

Example: *19.5 kilograms = ? milligrams*

> *Step 1:* *Figure out how many places to move the decimal. ... Henry Died By Drinking Chocolate Milk – that's six places. (Remember to count the one you are going to, but not the one you are on.)*
>
> *Step 2:* *Move the decimal six places to the right since you are going from larger to smaller. In this case you need to add zeros.*
>
> > *19.5 kilograms = 19,500,000 milligrams*

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

16. 1200 kilograms = ? milligrams
17. 14000 millimeters = ? meters
18. 670 hectometers = ? centimeters
19. 6544 liters = ? milliliters
20. .078 kilometers = ? meters
21. 17 grams = ? kilograms

Scientific Notation

Introduction:

Scientific notation is a shorthand way to express large or tiny numbers. Since you will need to do calculations throughout the year WITHOUT A CALCULATOR, we will consider anything over 1000 to be a large number. Writing these numbers in scientific notation will help you do your calculations much quicker and easier and will help prevent mistakes in conversions from one unit to another. Like the metric system, scientific notation is based on factors of 10. A large number written in scientific notation looks like this:

The number before the x (1.23) is called the <u>coefficient</u>. The coefficient must be greater than 1 and less than 10. The number after the x is the base number and is always 10. The number in superscript (11) is the <u>exponent</u>.

Part I: Writing Numbers in Scientific Notation

To write a large number in scientific notation, put a decimal after the first digit. Count the number of digits after the decimal you just wrote in. This will be the exponent. Drop any zeros so that the coefficient contains as few digits as possible.

> *Example:* 123,000,000,000
>
> > *Step 1:* *Place a decimal after the first digit.* 1.23000000000
> >
> > *Step 2:* *Count the digits after the decimal...there are 11.*
> >
> > *Step 3:* *Drop the zeros and write in the exponent.* 1.23×10^{11}

Writing tiny numbers in scientific notation is similar. The only difference is the decimal is moved to the left and the exponent is a negative. A tiny number written in scientific notation looks like this:

$$4.26 \times 10^{-8}$$

To write a tiny number in scientific notation, move the decimal after the first digit that is not a zero. Count the number of digits before the decimal you just wrote in. This will be the exponent as a negative. Drop any zeros before or after the decimal.

> *Example*: .0000000426
>> *Step 1*: 00000004.26
>> *Step 2*: Count the digits before the decimal...there are 8.
>> *Step 3*: Drop the zeros and write in the exponent as a negative. 4.26×10^{-8}

Part II: Adding and Subtracting Numbers in Scientific Notation

To <u>add</u> or <u>subtract</u> two numbers with exponents, the exponents must be the same. You can do this by moving the decimal one way or another to get the exponents the same. Once the exponents are the same, add (if it's an addition problem) or subtract (if it's a subtraction problem) the coefficients just as you would any regular addition problem (review the previous section about decimals if you need to). The exponent will stay the same. Make sure your answer has only one digit before the decimal – you may need to change the exponent of the answer.

> *Example*: $1.35 \times 10^6 + 3.72 \times 10^5 = ?$
>> *Step 1*: Make sure both exponents are the same. It's usually easier to go with the larger exponent so you don't have to change the exponent in your answer, so let's make both exponents 6 for this problem.
>>> $3.72 \times 10^5 \rightarrow .372 \times 10^6$
>> *Step 2*: Add the coefficients just as you would regular decimals. Remember to line up the decimals.
>>>
>>> $$1.35$$
>>> $$+\ .372$$
>>> $$\overline{1.722}$$
>> *Step 3*: Write your answer including the exponent, which is the same as what you started with.
>>> 1.722×10^6

Part III: Multiplying and Dividing Numbers in Scientific Notation

To <u>multiply</u> exponents, multiply the coefficients just as you would regular decimals. Then add the exponents to each other. The exponents DO NOT have to be the same.

> *Example*: 1.35×10^6 X $3.72 \times 10^5 = ?$
>> *Step 1*: Multiply the coefficients.
>>>
>>> $$1.35$$
>>> $$\times\ 3.72$$
>>> $$\overline{270}$$
>>> $$9450$$
>>> $$\underline{40500}$$
>>> $$50220 \rightarrow 5.022$$
>> *Step 2*: Add the exponents.
>>> $5 + 6 = 11$
>> *Step 3*: Write your final answer.
>>> 5.022×10^{11}

To <u>divide</u> exponents, divide the coefficients just as you would regular decimals, then subtract the exponents. In some cases, you may end up with a negative exponent.

Example: $5.635 \times 10^3 / 2.45 \times 10^6 = ?$
   Step 1: Divide the coefficients.

$$5.635 / 3.45 = 2.3$$

   Step 2: Subtract the exponents.

$$3 - 6 = -3$$

   Step 3: Write your final answer.

$$2.3 \times 10^{-3}$$

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

Write the following numbers in scientific notation:
22. 145,000,000,000
23. .000348
24. 135 trillion
25. 24 thousand

Complete the following calculations:
26. $3 \times 10^3 + 4 \times 10^3$
27. $7.89 \times 10^{-6} + 2.35 \times 10^{-8}$
28. $1.278 \times 10^{-13} - 1.021 \times 10^{-10}$
29. three hundred thousand plus forty-seven thousand
30. $3.78 \times 10^3 \times 2.9 \times 10^2$
31. three million times eighteen thousand
32. $1.98 \times 10^{-4} / 1.72 \times 10^{-6}$
33. twelve thousand divided by four thousand

Dimensional Analysis

Introduction
Dimensional analysis is a way to convert a quantity given in one unit to an equal quantity of another unit by lining up all the known values and multiplying. It is sometimes called factor-labeling. The best way to start a factor-labeling problem is by using what you already know. In some cases you may use more steps than a classmate to find the same answer, but it doesn't matter. Use what you know, even if the problem goes all the way across the page!
In a dimensional analysis problem, start with your given value and unit and then work toward your desired unit by writing equal values side by side. Remember you want to cancel each of the intermediate units. To cancel a unit on the top part of the problem, you have to get the unit on the bottom. Likewise, to cancel a unit that appears on the bottom part of the problem, you have to write it in on the top.
Once you have the problem written out, multiply across the top and bottom and then divide the top by the bottom.

Example: 3 years = ? seconds   *Step 1: Start with the value and unit you are given. There may or may not be a number on the bottom.*

$$\left[\quad \underline{3 \text{ years}} \quad \right]$$

Step 2: Start writing in all the values you know, making sure you can cancel top and bottom. Since you have years on top right now, you need to put years on the bottom in the next segment. Keep going, canceling units as you go, until you end up with the unit you want (in this case seconds) on the top.

$$\left[\frac{\cancel{3 \text{ years}}}{} \right] \left[\frac{365 \cancel{\text{ days}}}{1 \cancel{\text{ year}}} \right] \left[\frac{24 \cancel{\text{ hours}}}{1 \cancel{\text{ day}}} \right] \left[\frac{60 \cancel{\text{ minutes}}}{1 \cancel{\text{ hour}}} \right] \left[\frac{60 \text{ seconds}}{1 \cancel{\text{ minute}}} \right]$$

Step 3: Multiply all the values across the top. Write in scientific notation if it's a large number. Write units on your answer.

$$3 \times 365 \times 24 \times 60 \times 60 = 9.46 \times 10^7 \text{ seconds}$$

Step 4: Multiply all the values across the bottom. Write in scientific notation if it's a large number. Write units on your answer if there are any. In this case everything was cancelled so there are no units.

$$1 \times 1 \times 1 \times 1 = 1$$

Step 5: Divide the top number by the bottom number. Remember to include units.

$$9.46 \times 10^7 \text{ seconds} / 1 = 9.46 \times 10^7 \text{ seconds}$$

Step 6: Review your answer to see if it makes sense. 9.46×10^7 is a really big number. Does it make sense for there to be a lot of seconds in three years? YES! If you had gotten a tiny number, then you would need to go back and check for mistakes.

In lots of APES problems, you will need to convert both the top and bottom unit. Don't panic! Just convert the top one first and then the bottom.

Example: 50 miles per hour = ? feet per second

Step 1: Start with the value and units you are given. In this case there is a unit on top and on bottom.

$$\left[\frac{50 \text{ miles}}{1 \text{ hour}} \right]$$

Step 2: Convert miles to feet first.

$$\left[\frac{50 \cancel{\text{ miles}}}{1 \text{ hour}} \right] \left[\frac{5280 \text{ feet}}{1 \cancel{\text{ mile}}} \right]$$

Step 3: Continue the problem by converting hours to seconds.

$$\left[\frac{50 \cancel{\text{ miles}}}{1 \cancel{\text{ hour}}} \right] \left[\frac{5280 \text{ feet}}{1 \cancel{\text{ mile}}} \right] \left[\frac{1 \cancel{\text{ hour}}}{60 \cancel{\text{ minutes}}} \right] \left[\frac{1 \cancel{\text{ minute}}}{60 \text{ seconds}} \right]$$

Step 4: Multiply across the top and bottom. Divide the top by the bottom. Be sure to include units on each step. Use scientific notation for large numbers.

50 x 5280 feet x 1 x 1 = 264000 feet

1 x 1 x 60 x 60 seconds = 3600 seconds

264000 feet / 3600 seconds = 73.33 feet/second

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet. Use scientific notation when appropriate.

Conversions:

1 square mile = 640 acres

1 hectare (Ha) = 2.47 acres

1 kw-hr = 3,413 BTUs

1 barrel of oil = 159 liters

1 metric ton = 1000 kg

34. 134 miles = ? inches
35. 8.9×10^5 tons = ? ounces
36. 1.35 kilometers per second = ? miles per hour
37. A city that uses ten billion BTUs of energy each month is using how many kilowatt-hours of energy?
38. A 340 million square mile forest is how many hectares?
39. If one barrel of crude oil provides six million BTUs of energy, how many BTUs of energy will one liter of crude oil provide?
40. Fifty eight thousand kilograms of solid waste is equivalent to how many metric tons?

AP Environmental Science Graph Prep

Practice Interpreting Data:

The following questions are to help you practice reading information shown on a graph. Answer each question on the separate answer sheet.

1. Identify the graph that matches each of the following stories:
   a. I had just left home when I realized I had forgotten my books so I went back to pick them up.
   b. Things went fine until I had a flat tire.
   c. I started out calmly, but sped up when I realized I was going to be late.
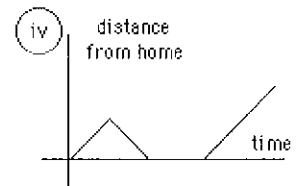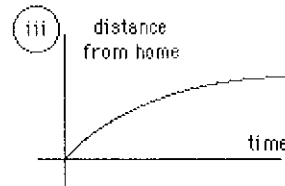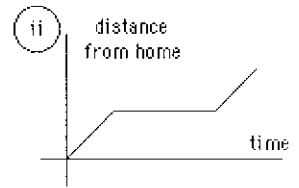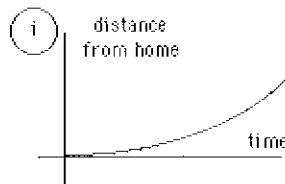
2. The
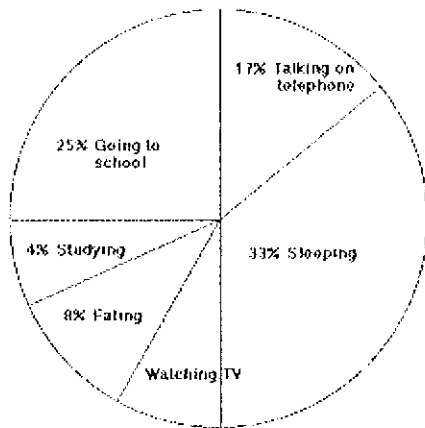
graph at the right represents the typical day of a teenager. Answer these questions:
   a. What percent of the day is spent watching TV?
   b. How many hours are spent sleeping?
   c. What activity takes up the least amount of time?
   d. What activity takes up a quarter of the day?
   e. What two activities take up 50% of the day?
   f. What two activities take up 25% of the day?

3. Answer these questions about the graph at the right:
   a. How many sets of data are represented?
   b. On approximately what calendar date does the graph begin?
   c. In what month does the graph reach its highest point?
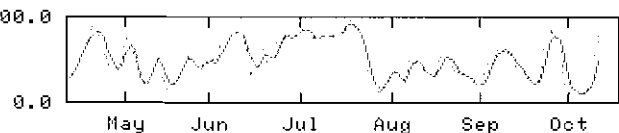
4. Answer these questions about the graph on the right:
   a. How many total miles did the car travel?
   b. What was the average speed of the car for the trip?
   c. Describe the motion of the car between hours 5 and 12?
   d. What direction is represented by line CD?
   e. How many miles were traveled in the first two hours of the trip?
   f. Which line represents the fastest speed?

A Trip By Car

Miles

200

150

100

50

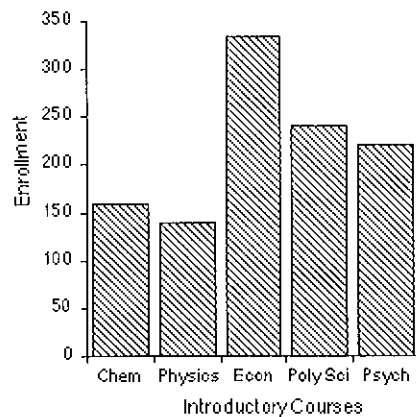0   5   10   15   20   25

Hours
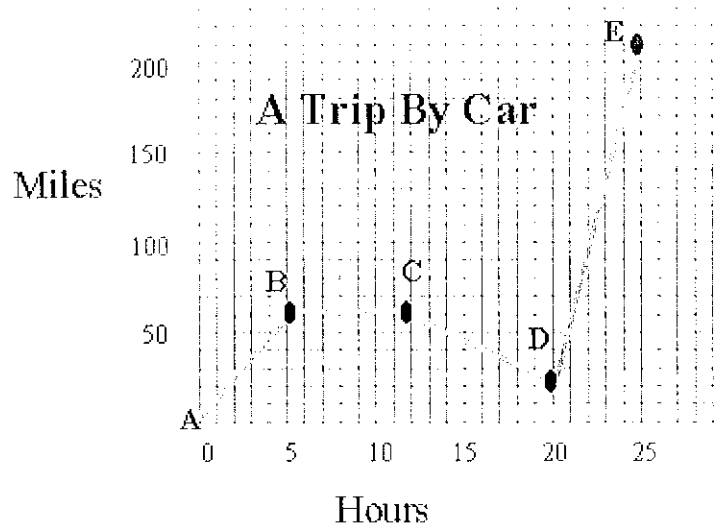
5. Answer these questions about the graph at the right:
   a. What is the dependent variable on this graph?
   b. Does the price per bushel always increase with demand?
   c. What is the demand when the price is 5$ per bushel?

Quantity Demanded (per week)
80
70
60
50
40
30
20
10
0

$1 $2 $3 $4 $5
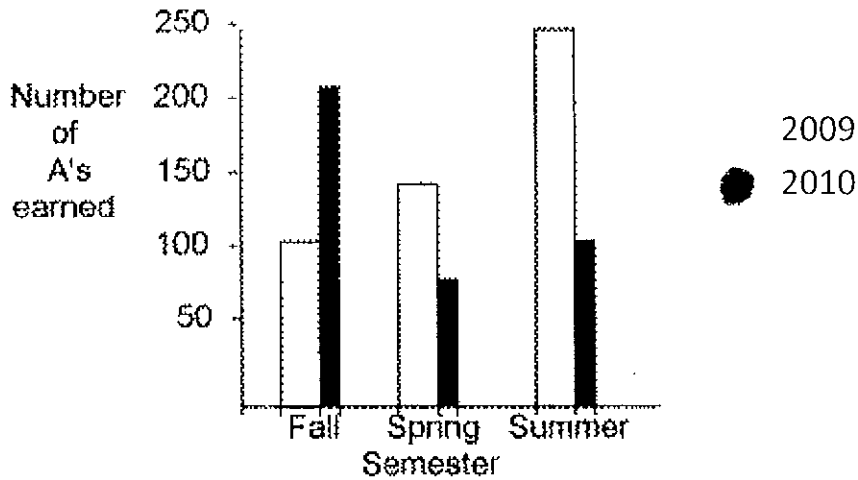Price per Bushel (dollars)
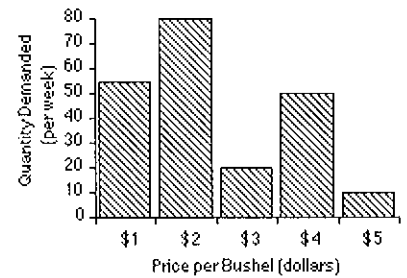
6. The bar graph below represents the declared majors of freshman enrolling at a university. Answer the following questions:
   a. What is the total freshman enrollment of the college?
   b. What percent of the students are majoring in physics?
   c. How many students are majoring in economics?
   d. How many more students major in poly sci than in psych?

Enrollment
350
300
250
200
150
100
50
0

Chem   Physics   Econ   Poly Sci   Psych
Introductory Courses

7. This graph represents the number of A's earned in a particular college algebra class. Answer the following questions:
   a. How many A's were earned during the fall and spring of 2009?
   b. How many more A's were earned in the fall of 2010 than in the spring of 2010?
   c. In which year were the most A's earned?
   d. In which semester were the most A's earned?
   e. In which semester and year were the fewest A's earned?

250

Number 200
of
A's 150
earned
100

50

Fall Spring Summer
Semester

● 2009
● 2010

8. Answer these questions about the graph below:
   a. How much rain fell in Mar of 1989?
   b. How much more rain fell in Feb of 1990 than in Feb of 1989?
   c. Which year had the most rainfall?
   d. What is the wettest month on the graph?

Rainfall 8
In
Inches 6

4

2

Jan Feb Mar Apr May June

1990

1989
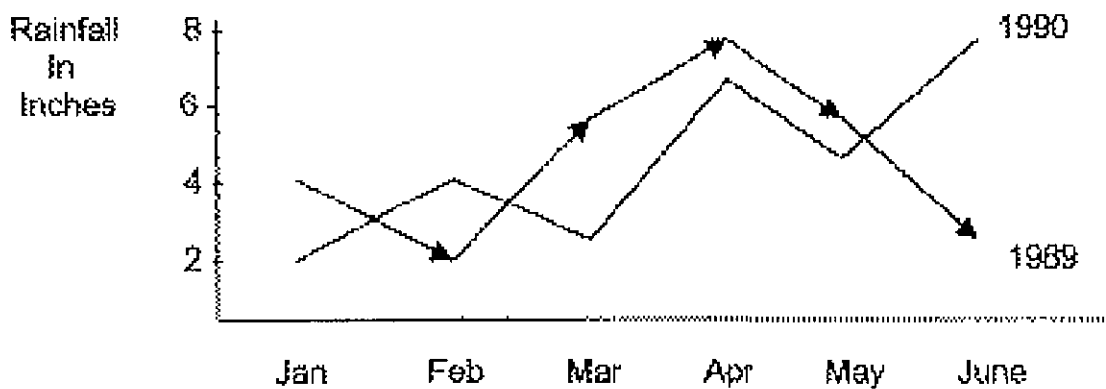
9. Answer these questions about the data table:

a. What is the independent variable on this table?
b. What is the dependent variable on this table?
c. How many elements are represented on the table?
d. Which element has the highest ionization energy?
e. Describe the shape of the line graph that this data would produce?

Atomic Number	Ionization Energy (volts)
2	24.46
4	9.28
6	11.22
8	13.55
10	21.47

10. Answer the following using the data table below:

a. How many planets are represented?
b. How many moons are represented?
c. Which moon has the largest mass?
d. Which planet has a radius closest to that of Earth?
e. How many moons are larger than the planet Pluto?
f. Which of Jupiter's moons orbits closest to the planet?
g. Which planet is closest to Earth?

Practice Making Graphs:

Use the following steps to create graphs and answer questions for each of the problems below. All your work will go on the separate answer sheet.

1. *Identify the variables. The independent variable is controlled by the experimenter. The dependent variable changes as the independent variable changes. The independent variable will go on the X axis and the dependent on the Y axis.*
2. *Determine the variable range. Subtract the lowest data value from the highest data value.*
3. *Determine the scale of the graph. The graph should use as much of the available space as possible. Each line of the scale must go up in equal increments. For example, you can go 0, 5, 10, 15, 20, etc. but you cannot go 1, 3, 9, 34, 50, etc. Increments of 1, 2, 5, 10, or 100 are commonly used but you should use what works best for the given data.*
4. *Number and label each axis.*
5. *Plot the data. If there are multiple sets of data on one graph, use a different color for each.*
6. *Draw a smooth, best-fit line for each data set.*
7. *Title the graph. Titles should explain exactly what the graph is showing and are sometimes long. Don't be afraid of a long title!*
8. *Create a key to the graph if there is more than one set of data.*

Problem 1

Age of the tree in years	Average thickness of the annual rings in cm. Forest A	Average thickness of the annual rings in cm. Forest B
10	2.0	2.2
20	2.2	2.5
30	3.5	3.6
35	3.0	3.8
50	4.5	4.0
60	4.3	4.5

The thickness of the annual rings indicate what type of environmental situation was occurring at the time of its development. A thin ring, usually indicates a rough period of development. Lack of water, forest fires, or a major insect infestation. On the other hand, a thick ring indicates just the opposite.

A. Make a line graph of the data.
B. What is the dependent variable?
C. What is the independent variable?
D. What was the average thickness of the annual rings of 40 year old trees in Forest A?
E. Based on this data, what can you conclude about Forest A and Forest B?

Problem 2

pH of water	Number of tadpoles
8.0	45
7.5	69
7.0	78
6.5	88
6.0	43
5.5	23

A. Make a line graph of the data.
B. What is the dependent variable?
C. What is the independent variable?
D. What is the average pH in this experiment?
E. What is the average number of tadpoles per sample?
F. What is the optimum water pH for tadpole development?
G. Between what two pH readings is there the greatest change in tadpole number?
H. How many tadpoles would you expect to find in water with a pH reading of 5.0?

Problem 3

Amount of ethylene in ml/m^2	Wine sap Apples: Days to Maturity	Golden Apples: Days to Maturity	Gala Apples: Days to Maturity
10	14	14	15
15	12	12	13
20	11	9	10
25	10	7	9
30	8	7	8
35	8	7	7

Ethylene is a plant hormone that causes fruit to mature. The data above concerns the amount of time it takes for fruit to mature from the time of the first application of ethylene by spraying a field of trees.

A. Make a line graph of the data.
B. What is the dependent variable?
C. What is the independent variable?